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GENERALIZED SENIORITY IN THE PRESENCE OF STRONG SHELL EFFECTS

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Generalized seniority is shown to provide a useful variational formalism for alike-nucleon systems even in the presence of strong shell effects.

In this letter we discuss the applicability of a generalized seniority [1] variational formalism in the presence of strong shell (or subshell) effects. We show that contrary to the traditional view such a formalism can reproduce the discontinuities in two-nucleon separation energies that arise when shell effects dominate. The earliest discussions of generalized seniority focused on its consequences if it is an exact symmetry. As shown by Talmi [1], this occurs if the hamiltonian satisfies some very restrictive commutation relations with the operators that create the energetically lowest two-particle eigenstates. In such cases, the ground state energy for a system of n active nucleons (n even) is given by

$$E_n = a + nb + \tfrac{1}{2}n(n-1)c \,. \tag{1}$$

Such an expression provides a smooth dependence of two-nucleon separation energies on the number of active nucleons n.

Recently, interest in generalized seniority has been rekindled by the phenomenological success of the interacting boson model (IBM) [2]. Generalized seniority provides a natural microscopic framework in which to derive the IBM in vibrational or near-vibrational nuclei [3,4]. Here, the focus is not on generalized seniority as an exact symmetry but rather as an approximate symmetry from which a meaningful variational approach can be formulated [‡1].

In such an approach, the ground state of a system of $2N$ valence nucleons (of a given type) is described by a generalized seniority $v = 0$ variational ansatz of the form

$$|S^N\rangle = \eta (S^\dagger)^N |\tilde{0}\rangle \,, \tag{2}$$

where

$$S^\dagger = \sum_j \alpha_j S_j^\dagger \,, \tag{3}$$

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[‡1] In this context, generalized seniority and the broken pair approximation [5] are equivalent.

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$$S_j^\dagger = [(j+\tfrac{1}{2})/2]^{1/2}\,[a_j^\dagger a_j^\dagger]^{(0)}\,, \tag{4}$$

$a_j^\dagger$ creates a nucleon in orbit j, $|\tilde{0}\rangle$ denotes the inert core and η is a normalization constant. The coefficients α_j are treated as variational parameters, determined by minimizing the expectation value of the hamiltonian.

In cases in which generalized seniority is exactly conserved, the above approach leads to the exact ground state. Recently [6], it has been confirmed that such an approach provides a good approximation to the ground state even when generalized seniority is not an exact symmetry. However, this has only been confirmed for cases involving a single active major shell with relatively close-lying orbits. An interesting and still unresolved issue is whether generalized seniority can also provide an acceptable variational approximation to low-lying states in the presence of strong shell effects.

A characteristic feature to be expected when shell effects dominate is a discontinuity in two-nucleon separation energies at the "shell closure". Although such a feature cannot be reproduced when generaized seniority is an exact symmetry [see eq. (1)] it is not inconceivable that it could be reproduced in a variational description.

To address the usefulness of generalized seniority in the presence of shell effects, we have studied a schematic problem involving two shells, each degenerate but split with respect to one another. The active orbits are

(i) those from the 50–82 major shell (denoted shell 1), which we assume to be degenerate, and

(ii) the $1h_{9/2}$ orbit (denoted shell 2), which we assume to be at energy Δ above the orbits in shell 1.

We consider the problem of N active nucleon pairs interacting through a surface delta interaction (SDI) [7] with strength g = 0.2 MeV. We have carried out variational calculations for the generalized seniority ansatz (2)–(4) for several values of the shell splitting Δ and for several nuclei near the N = 16 closure of shell 1. The variational minimizations were achieved using the iterative diagonalization procedure of ref. [3]. The resulting ground state binding energies are presented in table 1, in the column denoted BE (v = 0). The relevant dimensionless parameter that determines whether we are in an interaction-dominated or a single-particle-splitting-dominated regime is $\Delta/\Omega g$ where $\Omega = (\Omega_1\Omega_2)^{1/2} \approx 9$. Thus, for Δ = 0 MeV the interaction dominates, for Δ = 2 MeV the interaction and single-particle splitting are of comparable importance and for Δ = 4 MeV the single-particle

Table 1
Results of generalized seniority calculations for schematic two-shell model.

N	Δ (MeV)	BE (v=0) (MeV)	$\langle H^2 - \langle H^2\rangle\rangle$ (MeV)			v = 4 contributions	
			total	$L=0$	$L\neq 0$	BE (MeV)	wave function (%)
14	0	58.80	0	0	0	0	0
15		63.00	0	0	0	0	0
16		67.20	0	0	0	0	0
17		71.40	0	0	0	0	0
18		75.60	0	0	0	0	0
14	2	49.26	0.27	0.07	0.21	0.07	1.7
15		52.40	0.31	0.08	0.23	0.08	1.9
16		55.41	0.30	0.09	0.22	0.08	1.9
17		58.27	0.23	0.07	0.17	0.06	1.4
18		60.96	0.13	0.04	0.09	0.03	0.8
14	4	46.39	1.56	0.14	1.43	0.20	2.4
15		49.27	2.11	0.21	1.89	0.26	3.3
16		51.97	2.93	0.30	2.63	0.37	4.6
17		51.68	1.45	0.15	1.30	0.18	2.3
18		51.09	0.62	0.06	0.57	0.08	1.0

splitting (or shell effect) dominates.

The generalized seniority variational minimum is achieved when the trial wave function (2)–(4) is decoupled from all $v = 2$ states of the form $|S^{N-1}S'\rangle$, where S' corresponds to a $J = 0^+$ pair orthogonal to S. However, the hamiltonian can in principle couple the variational ground state to $v = 4$ states. To assess the significance of such generalized seniority-breaking effects, we have calculated the squared fluctuation energy, $\langle H^2 - \langle H\rangle^2\rangle$. This is also presented in table 1, in the column denoted $\langle H^2 - \langle H\rangle^2\rangle_{\mathrm{total}}$.

Finally, we have evaluated the contribution to the squared fluctuation energy arising from two types of intermediate states. The first, which we denote in the table by $\langle H^2 - \langle H\rangle^2\rangle_{L\neq 0}$, arises from intermediate states of the form $|S^{N-2}L_{11}L_{22}(0)\rangle$. Here, L_{ii} denotes a pair of particles in shell i coupled to angular momentum L. The second, which we denote by $\langle H^2 - \langle H\rangle^2\rangle_{L=0}$ arises from intermediate states of the form $|S^{N-2}(S')^2\rangle$. These two classes of intermediate states appear to exhaust the *full* squared fluctuation energy [‡2]. Since both involve the excitation of at least two particles from the correlated S pair into shell 2, we can obtain an upper limit to their contribution to the ground state energy and to their admixtures in the ground state wave function by using perturbation theory with an energy denominator 2Δ. These results are presented in the last two columns of the table.

In the limit of $\Delta = 0$, generalized seniority is exactly conserved and fluctuations in the ground state expectation value to identically to zero. What is remarkable, however, is that generalized seniority persists as an excellent approximation even for $\Delta = 4$ MeV, where shell effects are dominant and where naively one might have expected it to break down. Note that even in the worst case of $\Delta = 4$ MeV and $N = 16$, the ground state energy is obtained to within 0.37 MeV and $v = 4$ admixtures in the ground state are at most 4.6%. These are in fact generous upper limits since several of the $v = 4$ states that admix into the ground state are appreciably higher in energy than 2Δ.

Finally, in fig. 1, we plot the calculated two-nucleon separation energies [S_{2n} = BE (N) − BE($N-1$)] obtained in the generalized seniority approximation for the three values of Δ. For $\Delta = 0$ and 2 MeV, the two-nucleon separation energies vary smoothly with N. However, in the $\Delta = 4$ shell-effect-dominated regime, the generalized seniority approximation produces a discontinuity in S_{2n} at the $N = 16$ shell closure.

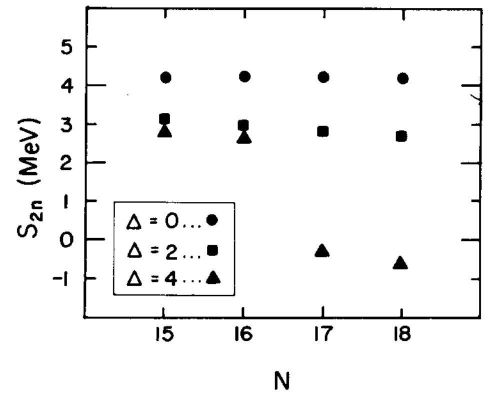

Fig. 1. Calculated two-nucleon separation energies S_{2n} as a function of the number of nucleon pairs N. Results are presented for three choices of the shell splitting Δ (given in MeV).

In conclusion, we have demonstrated that generalized seniority can provide a useful variational formalism even in the presence of strong shell or subshell effects. It provides an excellent approximation to both the ground state energy and the ground state wave function. Furthermore, in a regime in which shell effects dominate it is able to produce discontinuities in the two-nucleon separation energies.

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‡2 Other possible $v = 4$ states, e.g. $|S^{N-2}L_{11}L_{12}(0)\rangle$, do not seem to contribute.

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